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Quantum Fisher Information in the Generalized One-axis Twisting Model

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Abstract We investigate the quantum Fisher information (QFI) of symmetric states for spin-s particles. We derive the maximal QFI, and find that quantum spin correlations are essential ingredients of the maximal QFI. We make applications to the generalized one-axis twisting model. The results show that the redistributions of uncertainties on the basis of the quantum correlations in the multiqubit system are useful for sub-shot-noise phase sensitivity. Furthermore, for high-spin ($s > 1/2$) composite systems, we find a sufficient criterion for entanglement.

Keywords Quantum Fisher information · One-axis twisting model · Heisenberg limit · Entanglement

1 Introduction

Quantum entanglement plays an important role in many fields of quantum information processing, such as quantum teleportation [1], superdense coding [2], quantum key distribution [3], and telecloning [4]. In the past decades, various kinds of entanglement criteria were suggested. Quite recently, Pezzé and Smerzi [5] introduced a different criterion

$$\chi^2 = \frac{1}{\frac{F_Q[\rho_{\mathrm{in}}, S_{\vec{n}}]}{N}} < 1, \tag{1}$$

where N is the number of particles of the system under consideration, and F_Q is the QFI [6–9]. Thus the quantity F_Q/N gives the mean quantum Fisher information per particle, which is reciprocal to the quantity χ^2. In addition, ρ_{in} represents the input state, $S_{\vec{n}}$ is the

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collective operator defined as $S_{\vec{n}} \equiv S \cdot \vec{n} = \sum_{l=1}^{N} \sigma_l$ with σ_l a Pauli matrix operator on the lth particle, and $\vec{n}$ denotes the direction along which χ^2 is detected.

On the one hand, it is demonstrated that $\chi^2 \le \xi_{\mathrm{w}}^2$, where ξ_{w}^2 is the spin squeezing parameter following from Wineland's definition [10]. For a state, if $\xi_{\mathrm{w}}^2 < 1$, it is spin squeezed. Thus $\chi^2 < 1$ recognizes a class of states which are entangled, but not spin squeezed. This implies that as a criterion of entanglement, χ^2 takes an advantage over spin squeezing ξ_{w}^2. For instance, χ^2 can identify the Dicke state as entangled state, while ξ_{w}^2 can not.

On the other hand, the QFI is closely connected to the phase estimation in metrology and quantum sensors [11]. Generally, the output state is a linear rotation of the input state by an angle φ: $\rho_{\mathrm{out}} = e^{i\varphi S_{\vec{n}}} \rho_{\mathrm{in}} e^{-i\varphi S_{\vec{n}}}$. The estimation of the angle Φ is bounded by the shot-noise limit $\Delta\varphi_{\mathrm{SN}} \equiv 1/\sqrt{N}$. According to the Quantum Cramer-Rao theorem [7, 8], the phase sensitivity $\Delta\varphi$ has a lower bound limit

$$\Delta\varphi_{\mathrm{QCR}} = \frac{1}{\sqrt{F_Q[\rho_{\mathrm{in}}, S_{\vec{n}}]}} = \frac{\chi}{\sqrt{N}}. \tag{2}$$

If $\chi^2 < 1$, the state is entangled, and is useful for sub-shot-noise sensitivity of phase estimation. Especially, when $\chi = 1/\sqrt{N}$, the estimation sensitivity can reach the Heisenberg limit $1/N$. From (2), we see that the smaller χ (or bigger F_Q) is, the better of the input state for the phase estimation is. Therefore, in this work, we try to give a general expression of χ^2 (or F_Q) for the arbitrary symmetric pure states in systems composed of spin-s particles, and find its minimum (or maximum) over the whole coordinate space under certain condition.

This paper is organized as follows. In Sect. 2, under certain condition, we give a general expression of the minimum χ^2 (or MQFI) over the whole coordinate space. In Sect. 3, in terms of the general expression, we study the MQFI and its direction in the generalized one-axis twisting model. Finally, a conclusion is given in Sect. 4.

2 The Maximal Quantum Fisher Information

The expression of QFI in (1) is explicitly derived as [5, 12, 13]

$$F_Q[\rho_{\mathrm{in}}, S_{\vec{n}}] = 2\sum_{i,j} \frac{(p_i - p_j)^2}{p_i + p_j} |\langle\psi_j|S_{\vec{n}}|\psi_i\rangle|^2, \tag{3}$$

on the basis of $\rho_{\mathrm{in}} = \sum_i p_i|\psi_i\rangle\langle\psi_i|$ with p_i independent of the parameter Φ. For a pure state $\rho_{\mathrm{in}} = |\psi\rangle\langle\psi|$, the QFI is simplified as

$$F_Q[\rho_{\mathrm{in}}, S_{\vec{n}}] = 4(\Delta S_{\vec{n}})^2, \tag{4}$$

with $(\Delta S_{\vec{n}})^2 \equiv \langle S_{\vec{n}}^2\rangle - \langle S_{\vec{n}}\rangle^2$ the fluctuation of the collective operator $S_{\vec{n}}$.

A collection of N spin-s particles is represented by the collective operators

$$S_\alpha = \sum_{i=1}^{N} s_{i,\alpha}, \quad \alpha \in \{x, y, z\}, \tag{5}$$

where $s_{i,\alpha}$ is the spin operator for the ith particle. In the following, we will give a general expression of arbitrary symmetric states in such a composite system.

For convenience, we would like to construct a new coordinate frame such as [14, 15]

$$\begin{pmatrix} \vec{n}_1 \\ \vec{n}_2 \\ \vec{n}_3 \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\sin\phi & \cos\phi & 0 \\ -\cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta \end{pmatrix} \begin{pmatrix} \vec{n}_x \\ \vec{n}_y \\ \vec{n}_z \end{pmatrix}, \tag{6}$$

where $\vec{n}_1$ is the mean spin direction, $\vec{n}_2$ and $\vec{n}_3$ are the other two directions perpendicular to $\vec{n}_1$, θ and ϕ are polar and azimuth angle with their trigonometric functions

$$\begin{aligned} \cos\theta &= \langle S_z\rangle/\sqrt{\langle S_x\rangle^2+\langle S_y\rangle^2+\langle S_z\rangle^2}, \\ \sin\theta &= \sqrt{\langle S_x\rangle^2+\langle S_y\rangle^2}/\sqrt{\langle S_x\rangle^2+\langle S_y\rangle^2+\langle S_z\rangle^2}, \\ \tan\phi &= \langle S_y\rangle/\langle S_x\rangle. \end{aligned} \tag{7}$$

First, we will find the MQFI in the $(\vec{n}_2, \vec{n}_3)$ plane. For clarity, we denote $\vec{n}_\perp$ as an arbitrary direction in this plane, and introduce the collective operator $S_\perp$. According to [16], we can easily get the maximum fluctuation of $S_\perp$,

$$\max_{\vec{n}_\perp} \Delta S_\perp^2 = \frac{1}{2}\Big[\langle S_{\vec{n}_2}^2 + S_{\vec{n}_3}^2\rangle + \sqrt{\langle S_{\vec{n}_2}^2 - S_{\vec{n}_3}^2\rangle^2 + \langle [S_{\vec{n}_2}, S_{\vec{n}_3}]_+\rangle^2}\Big], \tag{8}$$

with

$$\begin{aligned} S_{\vec{n}_2} &= -\sin\phi S_x + \cos\phi S_y, \\ S_{\vec{n}_3} &= -\cos\theta\cos\phi S_x - \cos\theta\sin\phi S_y + \sin\theta S_z. \end{aligned} \tag{9}$$

In a special case that there is no spin correlation between the direction of $\vec{n}_2$ and $\vec{n}_3$, i.e., $\langle [S_{\vec{n}_2}, S_{\vec{n}_3}]_+\rangle^2 = 0$, we have a simple result

$$\max_{\vec{n}_\perp} \Delta S_\perp^2 = \max(\langle S_{\vec{n}_2}^2\rangle, \langle S_{\vec{n}_3}^2\rangle). \tag{10}$$

That is, the maximum spin fluctuation in the $(\vec{n}_2, \vec{n}_3)$ plane is just derived in the $\vec{n}_2$ direction or $\vec{n}_3$ direction. This situation exists in the LMG model [17, 18], and the anisotropic XY model in a transverse magnetic field [19–21]. Then combining (1) and (4), we obtain the MQFI and the minimum χ^2 in $(\vec{n}_2, \vec{n}_3)$ plane

$$\max_{\vec{n}_\perp} F_\perp = 4\max(\langle S_{\vec{n}_2}^2\rangle, \langle S_{\vec{n}_3}^2\rangle), \tag{11}$$

$$\min_{\vec{n}_\perp} \chi_\perp^2 = \frac{N}{4\max(\langle S_{\vec{n}_2}^2\rangle, \langle S_{\vec{n}_3}^2\rangle)}. \tag{12}$$

Then, we will find the MQFI over the whole coordinate space. We introduce an arbitrary spin collective operator $S_{(\Theta,\Phi)}$,

$$S_{(\Theta,\Phi)} = \sin\Theta\cos\Phi S_{\vec{n}_2} + \sin\Theta\sin\Phi S_{\vec{n}_3} + \cos\Theta S_{\vec{n}_1}, \tag{13}$$

with (Θ, Φ) an arbitrary direction in the total space. Therefore, the maximum fluctuation of $S_{(\Theta,\Phi)}$ can be obtained by finding the maximum value of $(\Delta S_{(\Theta,\Phi)})^2$ over the whole parameter space of Θ and Φ,

$$\max_{(\Theta,\Phi)} (\Delta S_{(\Theta,\Phi)})^2 = \max_{(\Theta,\Phi)} (\langle S_{(\Theta,\Phi)}^2\rangle - \langle S_{(\Theta,\Phi)}\rangle^2). \tag{14}$$

Generally, it is difficult to perform this calculation. However, for some especial states, such as the Dicke state $|n\rangle_J$, the even and odd states of operator $S_{\vec{n}_1}$, they have the following property

$$\langle S_\alpha \rangle = \langle S_\alpha S_{\vec{n}_1} \rangle = \langle S_{\vec{n}_1} S_\alpha \rangle = 0, \quad \alpha \in \{\vec{n}_2, \vec{n}_3\}. \tag{15}$$

Therefore, (14) can be simplified as

$$\max_{(\Theta,\Phi)} (\Delta S_{(\Theta,\Phi)})^2 = \max\Big(\Delta S_{\vec{n}_1}^2, \max_{\vec{n}_\perp} \Delta S_\perp^2\Big). \tag{16}$$

This implies that, under the condition of (15), the direction of MQFI over the whole space is confined in the mean spin direction $\vec{n}_1$ or its perpendicular plane $(\vec{n}_2, \vec{n}_3)$. Finally, the MQFI and the minimum χ^2 over the whole space take the form as

$$\max_{(\Theta,\Phi)} F = 4\max\Big(\Delta S_{\vec{n}_1}^2, \max_{\vec{n}_\perp} \Delta S_\perp^2\Big), \tag{17}$$

$$\min_{(\Theta,\Phi)} \chi^2 = \frac{N}{4\max(\Delta S_{\vec{n}_1}^2, \max_{\vec{n}_\perp} \Delta S_\perp^2)}. \tag{18}$$

The above expressions will be used to examine the states generated by the generalized one-axis twisting model in the next section.

In fact, the fluctuation $\Delta S_\perp^2$ can be expressed in terms of spin correlations

$$\begin{aligned}\max_{\vec{n}_\perp} \Delta S_\perp^2 = N \max_{\vec{n}_\perp} \Delta s_{i\perp}^2 + \frac{1}{2} N[(N-1)\mathrm{cov}(s_{i\perp}^+, s_{j\perp}^-) \\ + |(\Delta s_{i\perp}^-)^2 + (N-1)\mathrm{cov}(s_{i\perp}^-, s_{j\perp}^-)| - |(\Delta s_{i\perp}^-)^2|],\end{aligned} \tag{19}$$

where $s_{i\perp}^\pm = s_{i,\vec{n}_2} \pm i s_{i,\vec{n}_3}$, and $\mathrm{cov}(A, B) = \langle AB\rangle - \langle A\rangle\langle B\rangle$. Here and in the following, we set the indices $j \neq i$. In addition, the fluctuation of the mean spin operator $S_{\vec{n}_1}$ takes the form

$$\Delta S_{\vec{n}_1}^2 = N\langle s_{i,\vec{n}_1}^2\rangle + N(N-1)\langle s_{i,\vec{n}_1} s_{j,\vec{n}_1}\rangle - N^2\langle s_{i,\vec{n}_1}\rangle^2. \tag{20}$$

From (19) and (20), we see that if there is no spin correlation between different sites (i.e., the terms with $j \neq i$), the fluctuations $\max_{\vec{n}_\perp} \Delta S_\perp^2$ and $\Delta S_{\vec{n}_1}^2$ are only determined by the spin expectations of the same sites. Thus, the spin correlations between different sites, which leads to the redistribution of the fluctuations, is an essential ingredient of the MQFI. In addition, these two equations enable us to derive an analytical result of the generalized one-axis twisting model.

3 The Maximal Fisher Information in the Generalized One-axis Twisting Model

In order to generate spin squeezing in terms of the spin correlations, Kitagawa and Ueda introduced the so-called standard one-axis twisting model with Hamiltonian: $H_0 = 2\kappa S_z^2$ [22] where is spin $s = 1/2$. In our work, we generalize it to the spin-s case [23]

$$H = \kappa \sum_i^N s_{i,x}^2 + \lambda \sum_{i\neq j}^N s_{i,x} \otimes s_{j,x}. \tag{21}$$

The parameter κ and λ represent the spin interactions in the same and different sites, respectively. If $\lambda = \kappa$, this Hamiltonian reduces to the standard one-axis twisting model. For simplicity, we set $\kappa = 1$.

At the initial time, we suppose the multiparticle system is in a product state, i.e., $|S, -S\rangle$. According to the Hamiltonian (21), we can obtain the corresponding evolution state. Then we find that, first, the mean spin is always along the z direction during the time evolution of the system, and the other two directions $\vec{n}_2$ and $\vec{n}_3$ correspond to the x and y directions, respectively. The spin expectation value along z direction is given by

$$\langle s_{i,z}(t)\rangle = s\cos^{2s-1} t \cos^{2s(N-1)}(\lambda t). \tag{22}$$

Second, the condition (15) is satisfied, which indicates that we can use (18) to obtain the minimum χ^2 over the whole coordinate space. By a straightforward calculation, we get other spin correlations as

$$\begin{aligned}
\langle s_{i,x}^2(t)\rangle &= \frac{s}{2}, \\
\langle s_{i,y}^2(t)\rangle &= \frac{1}{2}\left\{s^2 + \frac{s}{2} - s\left(s - \frac{1}{2}\right)\cos^{2s-2}(2t)\cos^{2s(N-1)}(2\lambda t)\right\}, \\
\langle s_{i,x}(t)s_{j,x}(t)\rangle &= 0, \\
\langle s_{i,y}(t)s_{j,y}(t)\rangle &= \frac{s^2}{2}\{\cos^{4s-2}(\lambda t - t) - \cos^{4s-2}(\lambda t + t)\cos^{2s(N-2)}(2\lambda t)\}, \\
\langle s_{i,x}(t)s_{j,y}(t)\rangle &= s\cos^{2s-1} t \sin(\lambda t)\cos^{2S(N-1)-1}(\lambda t), \\
\langle s_{i,z}^2(t)\rangle &= \frac{1}{2}\left\{s^2 + \frac{s}{2} + s\left(s - \frac{1}{2}\right)\cos^{2s-2}(2t)\cos^{2s(N-1)}(2\lambda t)\right\}, \\
\langle s_{i,z}(t)s_{j,z}(t)\rangle &= \frac{s^2}{2}\{\cos^{4s-2}(\lambda t - t) + \cos^{4s-2}(\lambda t + t)\cos^{2s(N-2)}(2\lambda t)\}.
\end{aligned} \tag{23}$$

Combining (23), (20), (19) and (18), we can easily get the analytical result of the MQFI in the generalized one-axis twisting model. We will discuss the result in the cases of spin $s = 1/2$ and $s > 1/2$, respectively.

3.1 MQFI in Spin-1/2 System

Let us first study the dynamical evolution of the MQFI in multiqubit system. In this case $s = 1/2$, the Hamiltonian in (21) reduces to the standard one-axis twisting model [22]. The expectation values shown in (23) are consistent with those of [24]. We find that χ^2 is a periodic function of time with the period $T = \pi/\lambda$, which is determined by the strength of spin interaction λ. During the time evolution progress, χ^2 has a minimum value $1/N$ at the half period point $T/2$, at which the system is in GHZ states

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|S, S\rangle_{S_\alpha} - |S, -S\rangle_{S_\alpha}), \quad \alpha \in \{y, z\}. \tag{24}$$

Explicitly, the state is $|\mathrm{GHZ}\rangle_{S_z}$ for even N and $|\mathrm{GHZ}\rangle_{S_y}$ for odd N. In other words, at the time point $T/2$, the phase sensitivity of the state of the system can exactly reach the Heisenberg limit $1/N$ [11], as shown in Fig. 1. However, the best squeezing time is dependent on

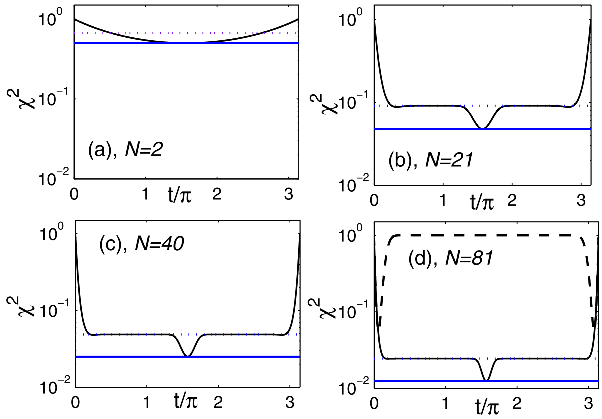

Fig. 1 (Color online) χ^2 (*black solid line*) and ξ_k^2 (*black dashed line*) versus time t for different parameter N. Parameters $\kappa = 1$, $\lambda = 1$. The lines $2/(N+1)$ (*blue dotted line*) and Heisenberg limit $1/N$ (*blue solid line*) are also plotted

the N, this implies that, in the general, the time at which χ^2 takes the minimum value do not correspond the best squeezing time (see Fig. 1(d)).

Next, we compare the ability checking entanglement of χ^2 and of the spin squeezing in the large N. Figure 1(d) shows that there is a wide time region, in which there is no spin squeezing, i.e., $\xi_w^2 \geq 1$ and $\xi_k^2 = 1$, where ξ_k^2 is the spin squeezing parameter following from Kitagawa's definition [22]. However, for the large N case, the fluctuations $\max_{\vec{n}_\perp} \Delta S_\perp^2$ and ΔS_z^2 can be approximatively given by

$$\begin{aligned}\max_{(x,y)} \Delta S_\perp^2 &= \frac{N}{8}[(N+1) - (N-1)\cos^{(N-2)}(2\lambda t)],\\ \Delta S_z^2 &= \frac{N}{8}[(N+1) + (N-1)\cos^{(N-2)}(2\lambda t)] - N^2\cos^{2(N-1)}(\lambda t)/4.\end{aligned} \tag{25}$$

These equations show that in this time region, χ^2 is maintained in

$$\frac{1}{N} \leq \chi^2 \leq \frac{2}{(N+1)}, \tag{26}$$

as shown in Fig. 1(b)–(d). This further proves that $\chi^2 < 1$ takes an advantage over the spin squeezing in detecting the entanglement of states. Furthermore, according to (2), we find

$$\Delta\varphi_{\mathrm{QCR}} \leq \frac{2}{\sqrt{N(N+1)}} \sim 1/N, \tag{27}$$

which attains the Heisenberg limit. From (25), this result suggest that when $\chi^2 \leq 2/(N+1)$, the quantum correlations among elementary spins are useful to essentially enhance phase estimation despite they can not create spin squeezing.

To further understand the above results, we plot the time evolution behaviors of the fluctuations ΔS_z^2 and $\max_{\vec{n}_\perp} \Delta S_\perp^2$ in Fig. 2. From Fig. 2(a) and (c), we see that, for even N, the maximal fluctuation $\max_{(\Theta,\Phi)}(\Delta S_{(\Theta,\Phi)})^2$ is not always in the (x, y) plane. At the point when $\Delta S_z^2 = \Delta S_\perp^2$, its direction turns to the z direction. However, for odd N, the maximal fluctuations are always in the (x, y) plane, and $\Delta S_\perp^2$ is just the maximal fluctuation $\max_{(\Theta,\Phi)}(\Delta S_{(\Theta,\Phi)})^2$ as shown in Fig. 2(b) and (d).

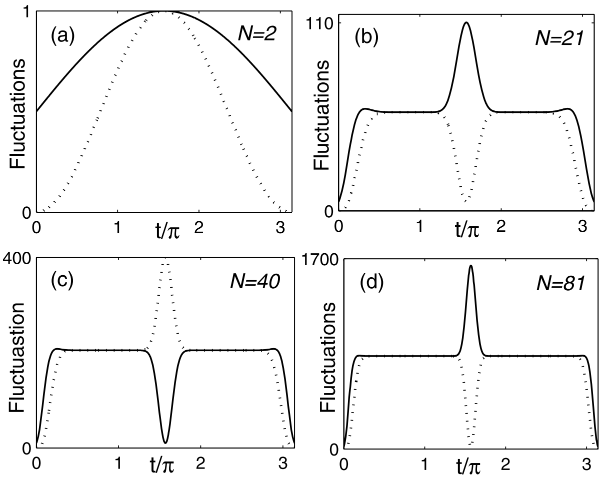

Fig. 2 The fluctuations $\Delta S_{\perp}^2$ (*black solid line*) and ΔS_z^2 (*black dashed line*) versus time t for different parameter N. Parameters $\kappa = 1$, $\lambda = 1$

3.2 MQFI in Spin-s System

Then we consider a general case $s > 1/2$. It was demonstrated that in the case of $s = 1/2$, the QFI is bounded by $N \le F_Q \le N^2$ [5]. If $F_Q > N$, then $\chi^2 < 1$, which indicates the multiqubit state is entangled. For the symmetric pure sates, this can be generalized to the $s > 1/2$ case as $2sN \le F_Q \le 4s^2N^2$. Thus, a question is put forward that if $F_Q > 2sN$, then whether $\chi_g^2 < 1/(2s)$ still implies entanglement or not for high-spin states. Here and in the following, we denote χ^2 in the global system as χ_g^2. To answer this question, it is necessary to calculate the χ^2 in the local system, denoted as χ_l^2, which can be obtained according to (3). For simplicity, we take the two-particle spin-2 system for example. In order to find the relations between χ_g^2, χ_l^2 and entanglement, we plot the dynamical behaviors of χ_g^2, χ_l^2 and the linear entropy for different strengths of interaction λ in Fig. 3.

When $\lambda = 0$, i.e., no spin interactions between different sites are involved, the global state is just a pure symmetric product state. Therefore, χ_g^2 is completely determined by the fluctuations of a single spin, i.e., $\chi_g^2 = \chi_l^2$. From Fig. 3(a), we note that $\chi_g^2 \le 1/(2s)$ always holds, and it reaches a minimum value $1/(4s^2)$ at the half period point $T/2$. However, there is no entanglement in the system as the linear entropy always equals to zero. This suggests that we can not check whether it is an entangled state or not from the inequality $\chi^2 < 1/(2s)$ for high-spin composite systems. In fact, it is easy demonstrated that $\chi_g^2 = \chi_l^2$ for the symmetric product state.

When $\lambda \ne 0$, we observe from Fig. 3(b) that the inequality $\chi_l^2 \ge \chi_g^2$ always holds, and when $\Delta\chi^2 \equiv \chi_l^2 - \chi_g^2 > 0$, the linear entropy $E_L > 0$ as well, which is different from the results of the case $\lambda = 0$. Therefore, it seems that

$$\Delta\chi^2 > 0 \tag{28}$$

may be regarded as an indicator of entanglement for the pure symmetric states in the high-spin composite systems.

To further confirm our findings, we consider another class of states, i.e., the entangled spin coherent states,

$$|\psi\rangle = C_1^{-1}(\cos\theta|\eta\rangle_1|\eta\rangle_2 + \sin\theta|-\eta\rangle_1|-\eta\rangle_2), \tag{29}$$

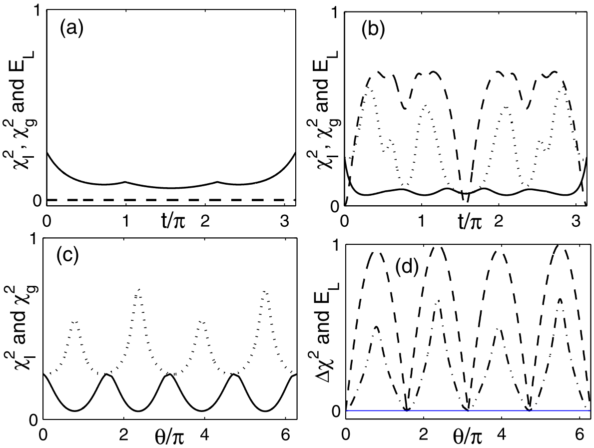

Fig. 3 (**a**) and (**b**) χ_l^2 (*dotted line*) and χ_g^2 (*solid line*) versus time t. Parameters $\kappa = 1$, $\lambda = 0, 4$, and $s = 2$ in (**a**) and (**b**). In (**c**) and (**d**), for the entangled spin coherent state, χ_l^2, χ_g^2, $\Delta\chi^2$ (*dashed dotted line*) and linear entropy E_L (*dashed line*) are plotted as a function of θ. Parameters: $\eta = 0.5$, $s = 2$. The linear entropy E_L is also plotted in (**a**) and (**b**)

where $C_1 = \sqrt{1 + \sin 2\theta\gamma^{4s}}$, $\gamma = (1 - |\eta|^2)/(1 + |\eta|^2) \in [0, 1]$, and $|\eta\rangle$ denotes the spin coherent state $|\eta\rangle = (1 + |\eta|^2)^{-s} \sum_{n=0}^{2s} \binom{2s}{n}^{1/2} \eta^n |n\rangle_s$, with the parameter η being complex. As a function of θ, in Fig. 3(c) and (d), we numerically compute χ_l^2, χ_g^2, and $\Delta\chi^2$ the linear entropy E_L. It can be seen that when $\Delta\chi^2 > 0$, linear entropy is greater than zero as well. Thus $\Delta\chi^2 > 0$ can also be regards as a criterion of entanglement.

4 Conclusion

In conclusion, we have studied the QFI for pure symmetric states. Under certain conditions, we derive a general expression for the MQFI over the whole coordinate space. The expression is applied to the generalized one-axis twisting model with particle spin s. In the case of $s = 1/2$, we explicitly discuss the MQFI and its direction. We find the MQFI has an maximal value $4N^2$ at the half period time point. For odd N, its direction is in the (x, y) plane, but for even N, it is not. In the case of arbitrary s, we give an inequality $2sN \le F_Q \le 4s^2N^2$ for the pure symmetric states. We find that $\Delta\chi^2$ can be regarded as an indicator of quantum entanglement in the high-spin composite systems.

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